FINITE-SIZE OR FINITE-DURATION EFFECTS IN THE MODELLING OF PHYSICAL SYSTEMS

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ABSTRACT

A common difficulty encountered by students of physics is in understanding the concept of models in Science. This problem, well documented in the educational research literature, is magnified by the fact that physical phenomena or processes that surround us have time discontinuities or geometrical boundaries. Let us mention, for instance, the time-evolution of a slowly-emptying deposit, the force exerted by a falling object of finite length, the charge/discharge of a capacitor, the electric and magnetic fields in finite or "infinite" systems, etc. It is usually claimed that the theoretical modeling and the experimental detection of effects due to the finiteness of the system under study is too hard to be undertaken in undergraduate (or even less, in high-school) teaching.

We shall show that the experimental measurement of these finite-size effects is possible and is, indeed, easy in many cases, and that much teaching profit can be gained from the qualitative analysis of the data, both at preuniversity and undergraduate teaching levels. A theoretical analysis will be provided also, in some cases.

SOME DIFFICULTIES ENCOUNTERED BY STUDENTS

In trying to explain phenomena observed in the physical world, physics has a long tradition in model building. Successful models with various degrees of sophistication (or, inversely, of idealization) are usually developed in different fields of physics. However, the students encounter difficulties with:

- the very concept of a "model" of a physical system,

- the assumptions of an infinitely long (or large, or wide ...) system,

- the concept of stationary versus transient effects (usually transient stages are absent in many chapters of academic physics).

We have in mind the electric and magnetic field of various sources with certain symmetries, electrical current phenomena, the discussion of friction in mechanics, etc. Even typical exemples of "finite" or transient phenomena become infinitely long in duration in the mathematical treatment: the modelling of the process of charging and discharging of a capacitor is a typical example. In other cases, simplifying assumptions are made that seem to go counter to first-hand experience. For instance, in the study of an emptying liquid reservoir, the velocity of the receding liquid/air surface is neglected. In many other situations finiteness is important: how does the force exerted by a falling chain change in character when all the chain links have fallen?; or, how does the force produced by an unwinding string coil evolve with time up to the point when all the string is unwound?

HOW TO DEAL WITH FINITENESS?

Although one may resort to, at least, three complementary approaches to deal with physical processes, namely, perform an experiment, a theoretical treatment, or a computer simulation, it turns out that each of these has its difficulties in dealing with finite time behavior or finite extent. Firstly, computer simulations tend to mask transient effects or, even worse, due to the usual recourse to random numbers in sequences of events, a simulation may make a stationary process to appear transient. Secondly, purely theoretical treatments, on its turn, are usually harder when transient time or finite size effects are investigated (just recall that the partial differential equations that describe the evolution of some systems cannot be simplified into total differential equations, or the difficulties in calculations of the fields due to finite sources of little symmetry). And thirdly, experimental measurements of fast or transient phenomena and of effects due to finite sources is usually rather laborious.

On the other hand, phenomena of a chaotic behavior are generally excluded in standard teaching, either because the mathematical treatment is too complex for an introductory level, or because well-known computer simulators (like the Working Model) do not reproduce them. An example is a sphere vibrating and oscillating in a spring from which it hangs. New teaching tools, like the Calculator Assisted Laboratory (CAL), may help us in studying experimentally the dynamics of this motion and of many other cases of finite systems mentioned above.

CAL: Calculator Assisted Laboratory

We have designed an introductory course on experimental science consisting of seminars, demonstrations and open experiments. The 40-hour course is based on a graphics calculator (some model from Texas Instruments, like the TI-82), various sensors and a device that collects experimental data automatically (a Calculator Based Laboratory or CBL). The equipment is easy to handle and allows carrying out experiments, both in high schools and introductory university courses, in a rather different way as compared with traditional laboratory courses. Since we have described this laboratory set up previously (Gras-Velázquez et al., 1999) we shall not describe it any further. Instead, we shall refer to a couple of examples of how CAL can be used to analyze finite systems.

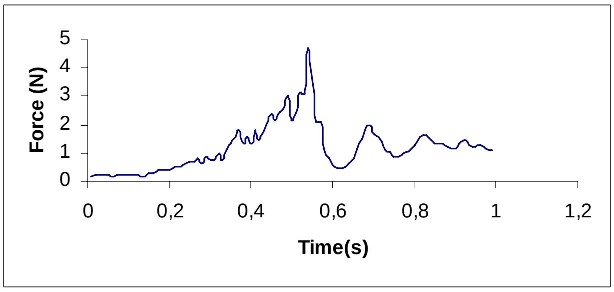

Fig.1: Time-dependence of the force exerted by a falling chain.

FORCE BY A FALLING CHAIN

As an example of the difficulties involved in the theoretical treatment of transient effects we shall discuss the force exerted upon a surface by a chain of mass *M* and length *L* falling freely, from the time it starts to fall until it finally rests upon the scale plate. Using CAL, we have obtained typically results as shown in fig.1. The theoretical treatment of this experiment by Bergen (1998) incorporates the gravitational and the impulse forces,

$$F = \frac{dp}{dt} = \frac{\partial p}{\partial x} v + \frac{\partial p}{\partial v} g$$

and the major part of the observations is well accounted for: the rising part of the curve is due to both terms (a quadratic time-dependent force up to a maximum 3Mg) whereas the “constant” rest value (Mg) of the weight-force for t > 0.7 s is reached when all the mass has fallen. The oscillations in F(t) are due to oscillations in the scale plate. However, an effect that is missing in the theory is the finiteness of the chain (Soler-Selva et al., 2000), that is the transient from the impulse-gravitational force to the pure gravitational component. In other words, the theory does not account for the sudden drop in the force that occurs at around 0.55 s. The full theoretical treatment involves the introduction of a step function in the momentum, $p = mv\,\Theta(L-x)$, to account for the finite length of the chain ($\Theta(u) = 1$, for $u > 0$, $\Theta(u) = 0$, for $u < 0$) and the subsequent appearance of a Dirac-delta, $\delta(u)$:

$$F = 3Mg\left[\frac{x}{L}\Theta(L-x)\right] - 2MgL\delta(L-x)$$

The first term is the one obtained by Bergen (1998) while the second term is due to the finite length of the chain, and accounts for the drop in F(t).

A FALLING MAGNET AND A COIL

With the CAL system one may investigate the potential induced in a coil by a magnet in free fall through it. In fig.2 we show a typical result.

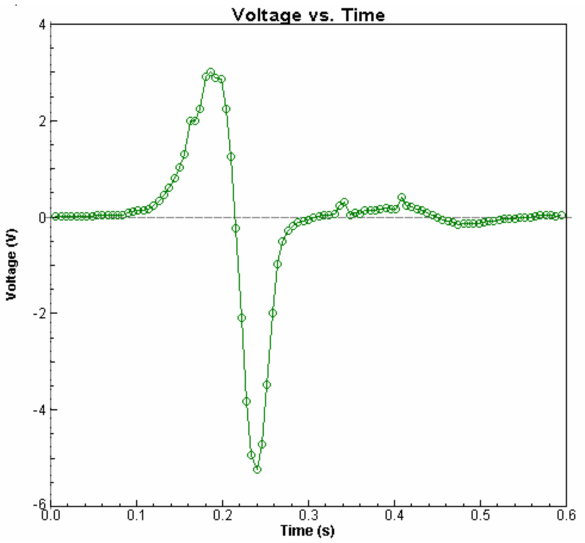

Fig.2: Voltage induced in a 42 Ω, 2000-turn coil by a cylindrical magnet falling through it.

It is easy to appreciate the usefulness of this experiment to conduct qualitative discussions in the classroom, both before and after having seen how the experiment is readily performed. Aside from a direct observation of finite-effects, one may consider Faraday and Lenz law, ac generators, etc. And still further, one may analyze effects due to relativity of motion (Huffman, 1980), by repeating the experiment with a fixed magnet and a falling coil.

After completion of this communication we have come across the following reference from Pico Technology (UK): http://www.picotech.com, where a similar experiment is proposed.

CONCLUSIONS

The qualitative and quantitative class discussion of examples like the ones mentioned above, both at undergraduate and graduate levels, are of great educational value. One may use them to emphasize basic aspects of the scientific endeavor like the advancement of assumptions, the design of the experiment, its performance, and the discussion of the results obtained. The use of CAL greatly facilitates the experimental part.

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